A new approach to solving boundary conditions in shallow water equations using a Riemann solver

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ABSTRACT: In recent years, the finite volume method has been improved by the Godunov method and Riemann solvers, but the method of characteristics (MOC) is generally used to introduce boundary conditions. This paper proposes a new approach to introducing boundary conditions in a way that is consistent with the Riemann problem solution. The same Riemann solver can be used for the entire domain, including the boundaries. This approach allows shocks and waves of any other type to be introduced as boundary conditions in a way that it is compatible with the Riemann problem solution.

1 INTRODUCTION

The use of the finite volume method in shallow water problems has become more widespread. The Godunov method and Riemann solvers have been combined to obtain high-resolution methods capable of solving transcritical regimes. To test the new advances, researchers usually work with typical standard problems such as dam breaks or bottom topography with a bump.

Problems of this type have boundary conditions that make mathematical sense, such as reflecting or non-reflecting boundary conditions. Under these conditions, the values of the dependent variables of the boundaries can be found immediately. Different boundary conditions are found in real computations applied to rivers and channels. In general (subcritical regimes), the boundary conditions are the upstream discharge value and the downstream water level. To apply boundary conditions, two upstream and two downstream dependent variables are needed. Therefore, the values of two free variables (upstream water level and downstream discharge) need to be interpolated.

This task is usually performed using the method of characteristics (MOC), but this involves using two different numerical schemes to solve the problem: a finite volume method inside the domain and the MOC at the boundaries. This solution is subject to the same problem as the MOC – it cannot be used to solve transcritical regimes. Thus, inside the domain we have a method that can deal with transcritical regimes, and at the boundaries we have a method that cannot deal with discontinuities.

This paper proposes a new approach to introducing normal hydraulic boundary conditions in a way that is consistent with a Riemann solver, making it unnecessary to use the MOC for the boundaries. With this method, boundary conditions with shocks and rarefactions can be used. This method determines which boundary conditions imply a well-posed hydraulic problem and which do not.

In a similar work (Bateman, 1993), the transcritical boundary conditions problem was solved using a modified MOC.

2 GODUNOV METHOD AND THE RIEMANN SOLVER IN SHALLOW WATER

The method proposed in this paper can be applied to any hyperbolic problem. To keep the solution scheme consistent, the same method must be used to solve the domain and the boundaries. This method was specially designed to work with a Godunov method (Godunov, 1959). It uses the Riemann problem solution structure, which is described next.

The Riemann problem solution has two different waves, one from the weak form and one from the quasilinear form. These two forms and their solutions are defined in the next section. The theory introduced in this chapter is discussed in many different publications (LeVeque, 2002), so we describe it only briefly.

Hyperbolic systems usually take the differential form, and we find differentials in the property and the flux.

$$\frac{\partial q(x,t)}{\partial t}+\frac{\partial f(q(x,t))}{\partial x}=0 \qquad (1)$$

Flux is $f\left(q(x,t)\right)$, $q(x,t)$ is a state in the state space, x and t are the independent variables, space and time. The rest of this paper deals with shallow water equations, which are systems of two hyperbolic equations that express mass and momentum conservation. There are two conservative dependent variables, depth h (m) and unit discharge hu (m^2/s):

$$q=\begin{pmatrix} h \\ hu \end{pmatrix}$$

$$\begin{cases} \dfrac{\partial h}{\partial t}+\dfrac{\partial hu}{\partial x}=0 \\ \dfrac{\partial hu}{\partial t}+\dfrac{\partial\left(u^2h+g\dfrac{h^2}{2}\right)}{\partial x}=0 \end{cases} \qquad (2)$$

Gravity is g, u is the velocity (m/s) defined as hu/h. These equations reproduce the behavior of a rectangular cross section of a channel without friction or slope.

2.1 *Flux computing and discretization*

The relationship between the Godunov method and the Riemann problem is due to the computation of the flux in (1). We have one flux at x_1, upstream of the control volume $[x_1,x_2]$, and the other at x_2, downstream of the control volume. The Godunov method computes these fluxes by solving a Riemann problem. A Riemann problem is a differential problem that has a discontinuity with dependent variable values on either side of it. We can define the average flux between two control volumes at this time as $F_{1+1/2}^{n}$, and use the self similarity of the hyperbolic equations to compute it.

$$F_{i+1/2}^{n}=\frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f\left(q(x_{i+1/2},t)\right)dt$$

$$F_{i+1/2}^{n}=f\left(q(x_{i+1/2},t)\right)\frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} dt$$

$$F_{i+1/2}^{n}=f\left(q(x_{i+1/2},t)\right) \qquad (3)$$

$$F_{i+1/2}^{n}=f(q^{\perp}(Q_i^n,Q_{i+1}^n))$$

$$F_{i+1/2}^{n}=f(q^{\perp}(q_l,q_r))$$

Finally, the Riemann problem solves the hyperbolic equation system between states q_l,q_r, to find the flux solution state $q^{\perp}(q_l,q_r)$. The boundary conditions are an exception because the dependent variables on either side of the Riemann problem are unknown. For example, in the downstream boundary, the right state q_r is known and the left state q_l has one unknown dependent variable (subcritical regime).

2.2 *Riemann Solver*

In the SWE Riemann problem there are two families of solution waves, the Rankine Hugoniot waves and the Riemann invariant waves, the first ones are a discontinous waves, namely shocks and the second one are continous waves, namely rarefactions. The solution consists of two states q_l,q_r connected to a third state q_m by waves. Thus, a wave connects q_l,q_m and a wave connects q_m,q_r. Every wave must belong to a family of curves. Each family of solutions has two waves, thus, in every state in the state space, we find four different waves: *Shock* λ_1, *Shock* λ_2, *Riemann* λ_1, *Riemann* λ_1 and *Riemann* λ_2. However, only some parts of these waves are valid. This condition is imposed by entropy (Lax, 1972).

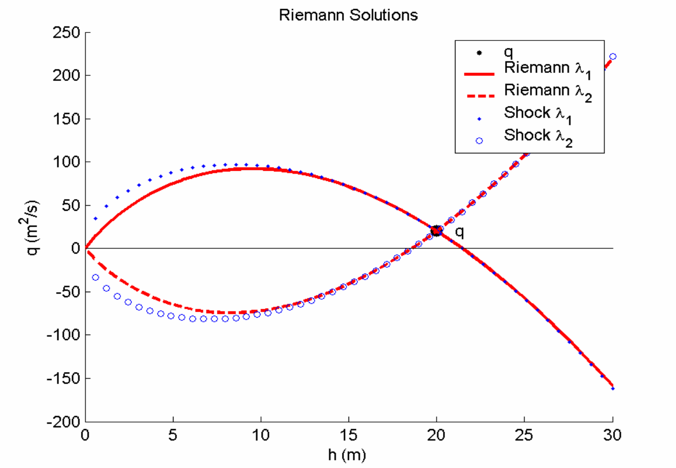

Figure 1. 4 Avaible curves in the state space associated with one state q and solution to a Riemann problem.

Using the valid solutions from Lax (1972), we constructed a solver by combining the different valid waves to obtain a state q_m, which is connected to a 1-wave in the left state of the Riemann problem q_l and a 2-wave in the right state of the Riemann problem q_r. The valid waves that connect q_l to q_m are

$$u_m=\begin{cases} u_l+2\left(\sqrt{gh_l}-\sqrt{gh_m}\right) & h_m<h_l \\ u_l-\left(h_m-h_l\right)\sqrt{\dfrac{g}{2}\left(\dfrac{1}{h_m}+\dfrac{1}{h_l}\right)} & h_m>h_l \end{cases} \qquad (4)$$

The valid waves that connect q_r to q_m are

$$u_m=\begin{cases} u_r-2\left(\sqrt{gh_r}-\sqrt{gh_m}\right) & h_m<h_r \\ u_r-\left(h_m-h_r\right)\sqrt{\dfrac{g}{2}\left(\dfrac{1}{h_m}+\dfrac{1}{h_r}\right)} & h_m>h_r \end{cases} \qquad (5)$$

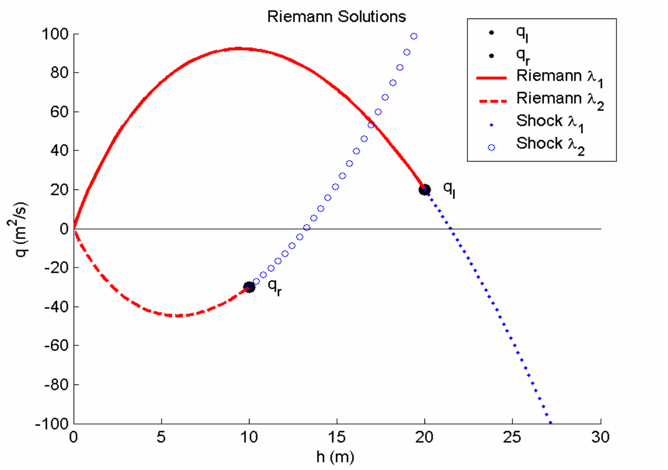

Figure 2. Valid solutions connecting with the two Riemann problem states q_l, q_r.

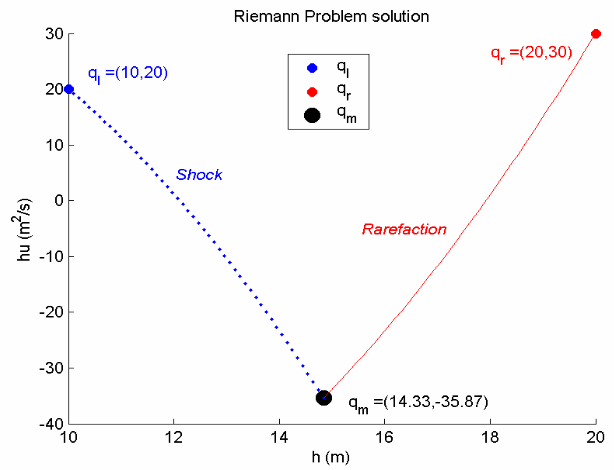

Figure 3. Example of a Riemann problem solution, the left state q_l is conneted to the solution state q_m by a *Shock* λ_1 and the solution state is connected to the right state q_r by a *Riemann* λ_2 wave.

It's important to note the difference between q_m and $q^{\perp}$, the first one is the Riemann solution and the second one is the flux solution. This difference is essential for the proposed algorithm to solving boundary conditions.

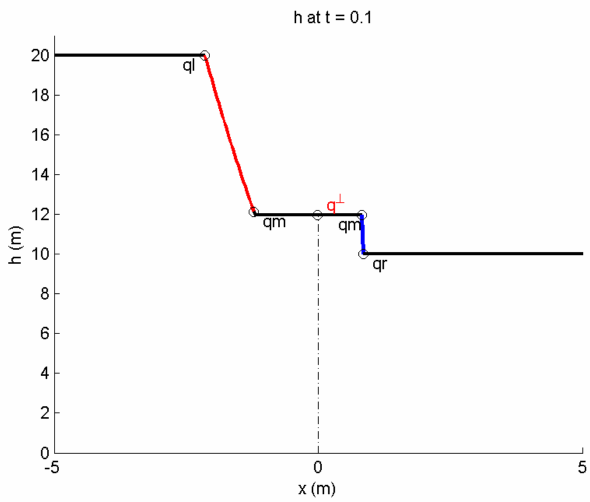

Figure 4. Example of the Riemann problem between q_l, q_r solution q_m and flux computation $q^{\perp}$.

Thus, the algorithm of the Godunov method is

$$\begin{gathered} \textit{Dependent Variables} \begin{cases} q_l = Q_i^n \\ q_r = Q_{i+1}^n \end{cases} \\ \downarrow \\ q_m = \textit{RiemannSolver}(q_l, q_r) \\ \downarrow \\ q^{\perp} = q^{\perp}(q_l, q_r, q_m) \\ \downarrow \\ F_{i+1/2}^n = f(q^{\perp}) \end{gathered} \tag{6}$$

3 BOUNDARY CONDITIONS

With hyperbolic problems, the definition of boundary conditions is not trivial. The number and application points of the conditions determine whether the problem is well posed. The shallow water equation system has two dependent variables and two equations. The system is first-order, so two boundary conditions and an initial state must be imposed. In the domain, there are two different boundaries $x_{1-1/2}, x_{n+1/2}$, and each boundary condition must be imposed on the right boundary.

3.1 *Description of the problem*

In hyperbolic problems, conditions should be imposed on the upwind side, which is defined by the eigenvalues of the hyperbolic system. They can be both positive, both negative, or one negative and one positive. These three cases are the supercritical regime, inverse regime and subcritical regime. In a supercritical regime, the boundary conditions should be defined on the upstream boundary. In an inverse regime, they should be imposed on the downstream boundary. In the subcritical regime, one condition should be imposed on the upstream boundary and the other on the downstream boundary.

As mentioned above, to solve a Riemann problem we need to know the values of the dependent variables on both sides of the Riemann problem. In the supercritical and subcritical regimes, all of the Riemann dependent variables of one boundary are defined and, on the other side, one dependent variable is defined and the other one is unknown. In these regimes, a complete boundary conditions method should test whether the Riemann problem solutions on the boundaries depend on the right dependent variables. On the boundary with all of the conditions imposed, the solution should be the conditions imposed. On the side without boundary conditions, the Riemann problem solution should not depend on the unknown outside dependent variables. This defines the well-posed problem condition.

The subcritical regime is different. In order to solve the Riemann problem, we have one boundary condition on each side, so the other one must be interpolated. A complete boundary conditions method should find the unknown dependent variable. It should be consistent with the imposed boundary condition and the domain dependent variables. The well-posed problem conditions used for the other regimes should also be applied.

First, the method should test the well-posed condition. Second, the method should interpolate the values of the incomplete dependent variables. These problems have traditionally been solved with the MOC, but this is not consistent with a Godunov method capable of dealing with transcritical regimes.

3.2 *General description of the solution*

This method uses the Riemann problem solution structure to introduce the boundary conditions in the domain. We will use the Riemann solution structure to test whether the boundaries are well posed, and we will use this structure to compute the values of the incomplete boundary conditions (subcritical).

For supercritical and inverse regimes, we need the complete boundary with two imposed conditions (h_{cc}, hu_{cc}) to belong to the supercritical or inverse zone of the state space. We also need the Riemann problem solution state q_m to fulfill another condition, depending on the regime, in order to obtain $q^{\perp} = (h_{cc}, hu_{cc})$.

For subcritical regimes, one dependent variable is imposed on each side of the domain and the other needs to be interpolated. To find this unknown variable, we will impose that it belong to the states connected by valid waves to the inside domain state. Figure 5 shows an example of a subcritical regime with a unit discharge hu_{cc} imposed in the upstream and the other dependent variable h_{cc} interpolated. The state (h_c, hu_{cc}) is connected to the inside state q_r by a *Riemann* λ_2 rarefaction.

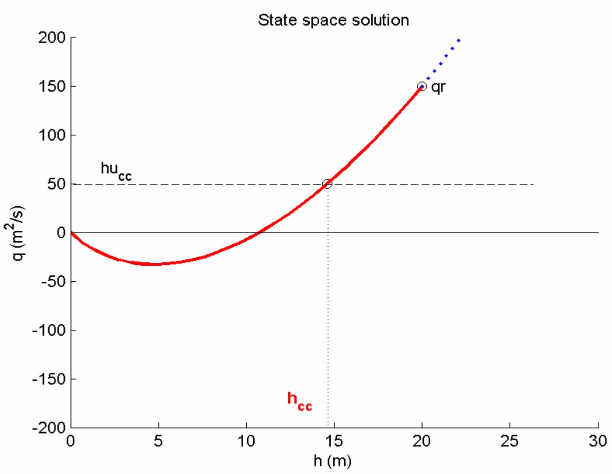

Figure 5. Deduction of the boundary condition h_{cc} from the inside state q_r and the imposed boundary condition hu_{cc}

$$q_r = Q_1^n$$
$$q_l = \begin{pmatrix} h? \\ hu_{cc} \end{pmatrix} \tag{7}$$

Conceptually, this method finds q_m, but for the Godunov method we need $q^{\perp}$. However, as we will see:

In a subcritical regime, boundary conditions are only well posed if $q^{\perp} = q_m$ is fulfilled.

The demonstration is simple. If we impose a unique boundary condition $(h_{cc}\ or\ hu_{cc})$, the resulting flux should depend on the inside state $(q_l\ or\ q_r)$ and on the boundary condition itself. The states that depend on the inside state $(q_l\ or\ q_r)$ have one degree of freedom, so by imposing the boundary conditions $(h_{cc}\ or\ hu_{cc})$, the value of the solution state q_m is defined exactly.

In some cases the solution is undefined, but these cases are not mathematically well posed. The flux solution $q^{\perp}$ should therefore fulfill $q^{\perp} \in [q_m, q_l]$ or $q^{\perp} \in [q_m, q_r]$, depending on the case (downstream or upstream conditions). In some cases, we will find $q^{\perp} = q_m$. However, other cases fulfill $q^{\perp} \in (q_m, q_l]$ or $q^{\perp} \in (q_m, q_r]$, such as transcritical rarefactions. In these cases, the problem is not well posed because $q^{\perp} \neq (h_{cc}, hu_{cc})$, so the boundary condition is not imposed.

3.3 *Figures used to describe the cases*

To illustrate different situations in which this method can be applied, we will use two different figures: one with the state space (Figure 7) and another with a dependent variable (h) in the space domain (Figure 8). The latter will reproduce the waves produced in the state space.

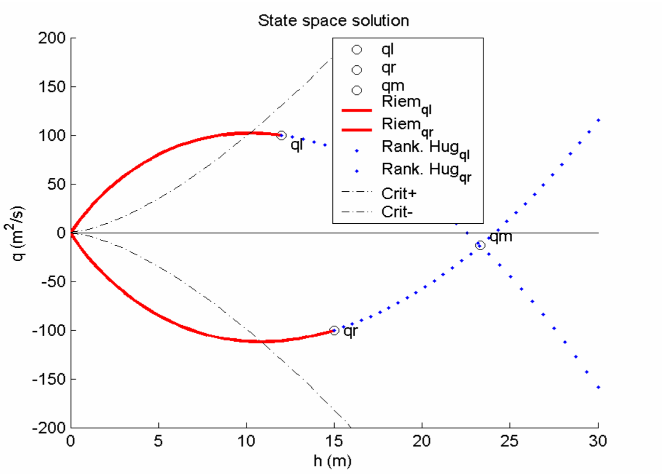

Figure 6. State space figure with the Riemann problem states, the solution state, the waves and the supercritical and inverse zones boundaries

The state space graphs will contain the Riemann problem states q_l, q_r, the solution state q_m and the flux state $q^{\perp}$. The valid waves connected to these states will also be drawn. The continuous waves represent the rarefactions; q_l connects to $Riemann\,\lambda_1$ and q_r connects to $Riemann\,\lambda_2$. The discontinuous lines represent the shocks; q_l connects to $Shock\,\lambda_1$ and q_r connects to $Shock\,\lambda_2$. We will also find two curves, Crit+ and Crit-, which define the boundaries of the supercritical and inverse regime zones respectively, λ_1, λ_2 are the eigenvalues of the hyperbolic system.

$Supercritical\ regime\ Crit+$

$$\lambda_1 = 0 \Rightarrow u - \sqrt{gh} = 0 \Rightarrow \tag{8}$$

$$u = \sqrt{gh} \Rightarrow \frac{u}{\sqrt{gh}} = 1 = Froude\,number$$

$Inverse\ regime\ Crit-$

$$\lambda_2 = 0 \Rightarrow u + \sqrt{gh} = 0 \Rightarrow \tag{9}$$

$$u = -\sqrt{gh} \Rightarrow \frac{-u}{\sqrt{gh}} = -1 = Froude\,number$$

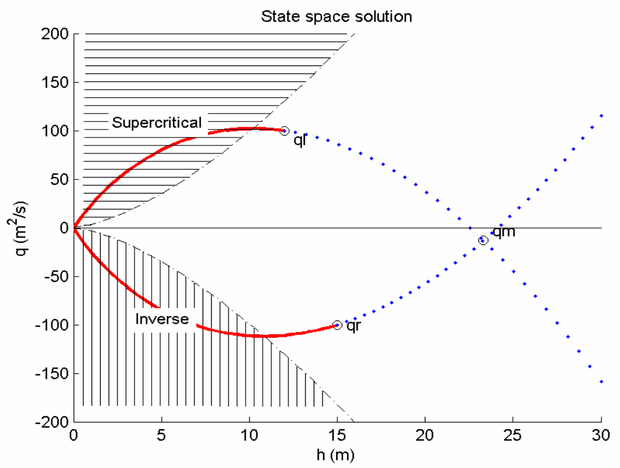

Figure 7. Inverse regime zone and supercrcitical zone. The remaining zone is for the subcritical regime

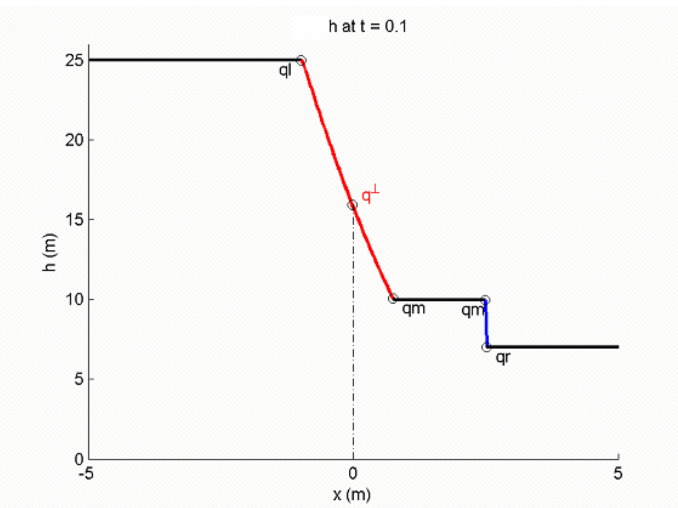

Figure 8. Dependent variable h in the space domain associated with the states $q_l, q_r, q_m, q^{\perp}$. The waves were developed for 0.1 s Possible cases

3.4 *Possible cases*

This section analyzes various cases and interprets them as boundary conditions. Each case has two interpretations: one as an upstream condition $(q_l, q_m, q^{\perp})$, only one wave connecting q_l, q_m, and the other as a downstream condition $(q_r, q_m, q^{\perp})$, only a wave connecting q_r, q_m. The name of the case depends on q_m.

3.4.1 *Subcritical regime*

In this case, we have $q^{\perp} = q_m$, so the state q_m imposed as a boundary condition for a subcritical regime is valid. If we tried to impose q_l as a supercritical condition on the upstream boundary, it would be wrong because $q^{\perp} \neq q_l$. Likewise, if we tried to impose q_r as an inverse regime boundary condition, it would be wrong because $q^{\perp} \neq q_r$.

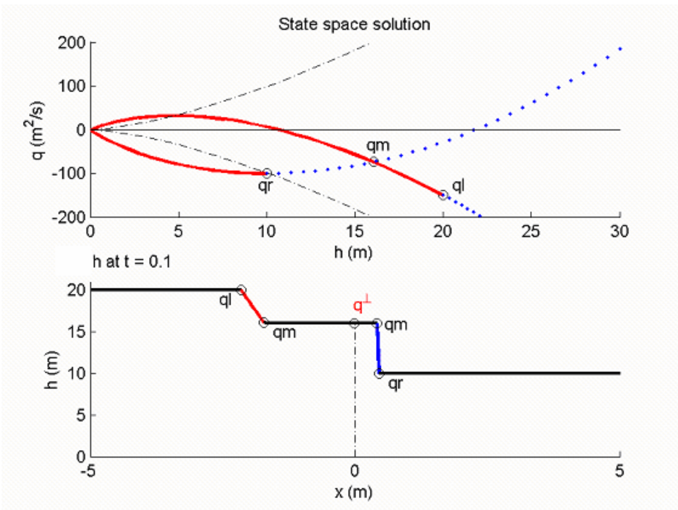

Figure 9. State space and waves

In the state space, a $Shock\,\lambda_2$ wave connects q_r to q_m and a $Riemann\,\lambda_1$ wave connects q_l to q_m.

3.4.2 *Subcritical regime*

In this case, we have $q^{\perp} = q_m$, so the state q_m imposed as a boundary condition for a subcritical regime is valid.

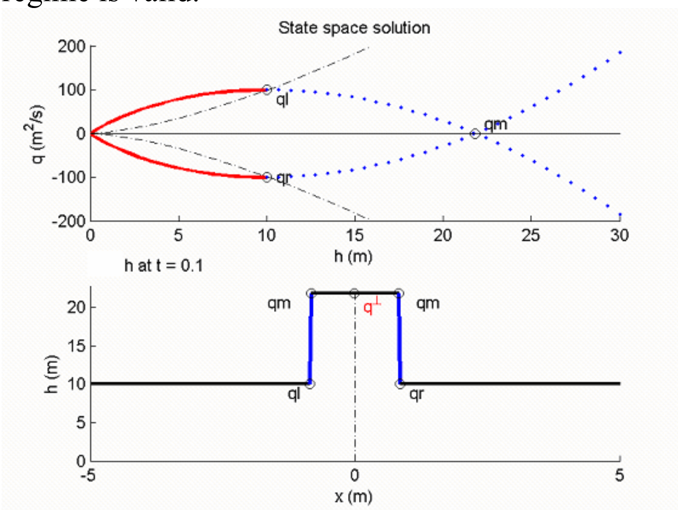

Figure 10 State space and waves

If we tried to impose q_l as a supercritical condition on the upstream boundary, it would be wrong because $q^{\perp} \neq q_l$. Likewise, if we tried to impose q_r as an inverse regime boundary condition, it would be wrong because $q^{\perp} \neq q_r$. In the state space, a *Shock* λ_2 wave connects q_r to q_m and a *Shock* λ_1 wave connects q_l to q_m.

3.4.3 *Subcritical regime*

In this case, we have $q^{\perp} = q_m$, so the state q_m imposed as a boundary condition for a subcritical regime is valid. If we tried to impose q_l as a supercritical condition on the upstream boundary, it would be wrong because $q^{\perp} \neq q_l$. Likewise, if we tried to impose q_r as an inverse regime boundary condition, it would be wrong because $q^{\perp} \neq q_r$.

In the state space, a *Shock* λ_1 wave connects q_r to q_m and a *Riemann* λ_2 wave connects q_l to q_m.

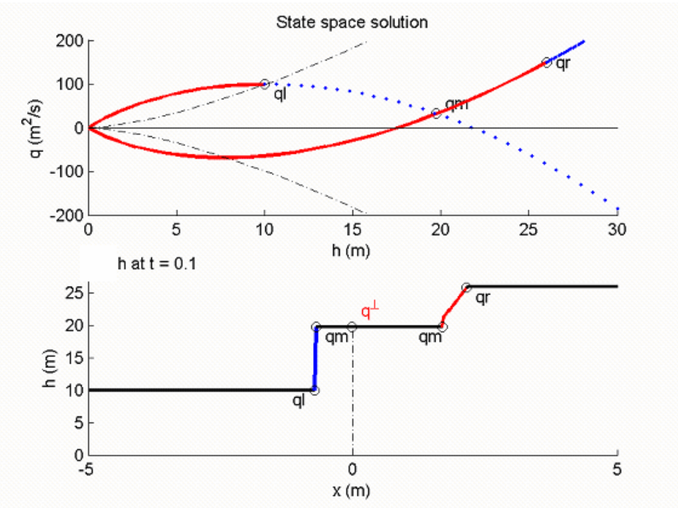

Figure 11. State space and waves

3.4.4 *Subcritical regime*

In this case, we have $q^{\perp} = q_m$, so the state q_m imposed as a boundary condition for a subcritical regime is valid.

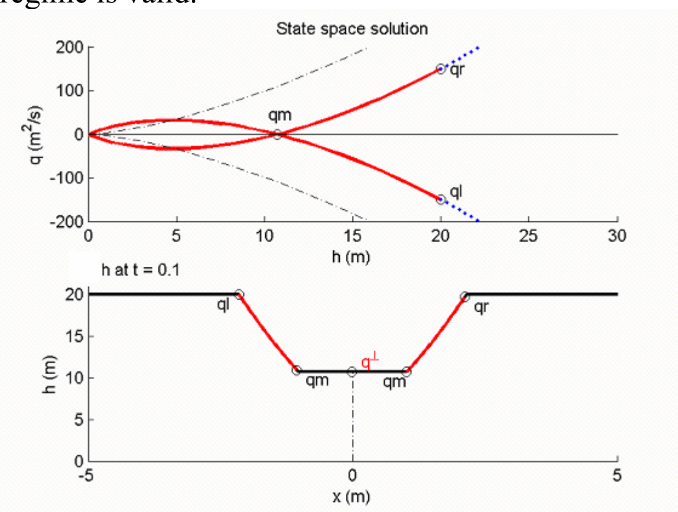

Figure 12. State space and waves.

If we tried to impose q_l as a supercritical condition on the upstream boundary, it would be wrong because $q^{\perp} \neq q_l$. Likewise, if we tried to impose q_r as an inverse regime boundary condition, it would be wrong because $q^{\perp} \neq q_r$.

In the state space, a *Riemann* λ_1 wave connects q_r to q_m and a *Riemann* λ_2 wave connects q_l to q_m.

3.4.5 *Inverse regime*

As a downstream condition in a subcritical regime (q_r does not exist), q_m would be valid because $q^{\perp} = q_m$. As a downstream inverse regime boundary condition ($q_r = (h_{cc}, hu_{cc})$), q_r would not be valid because $q^{\perp} \neq q_r$.

As an upstream boundary condition (supercritical or subcritical), it would be wrong because $q^{\perp} \neq q_l$ in supercritical ($q_l = (h_{cc}, hu_{cc})$) and $q^{\perp} \neq q_m$ in subcritical.

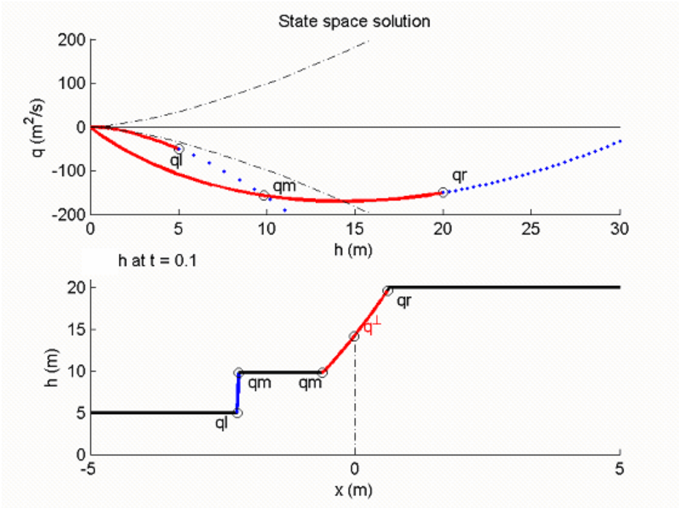

Figure 13. State space and waves

3.4.6 *Inverse regime*

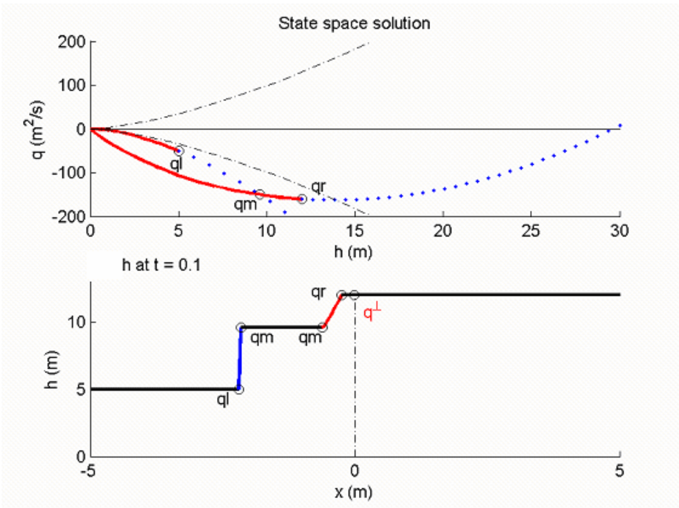

Figure 14. State space and waves.

As a downstream condition in a subcritical regime (q_r does not exist), q_m would be valid because $q^{\perp} = q_m$. As a downstream inverse regime boundary condition ($q_r = (h_{cc}, hu_{cc})$), it would be valid because $q^{\perp} = q_r$.

As an upstream boundary condition (supercritical or subcritical), it would be wrong because $q^{\perp} \neq q_l$ in supercritical ($q_l = (h_{cc}, hu_{cc})$) and $q^\perp \neq q_m$ in sub-critical.

3.4.7 *Inverse regime*

This case is interpreted like the previous one but has two shocks.

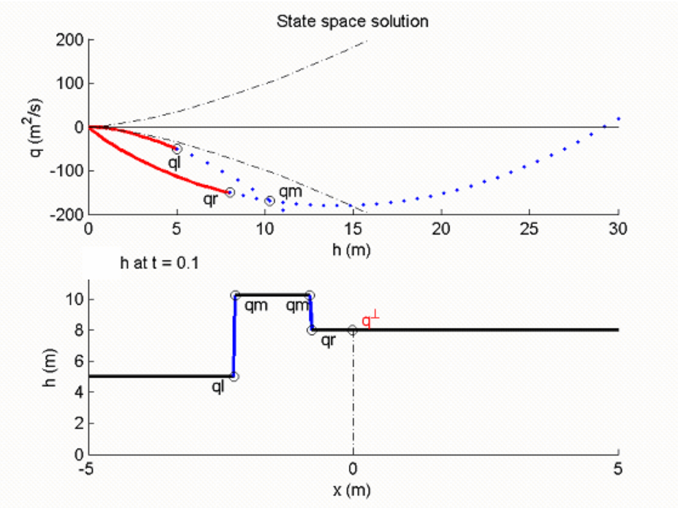

Figure 15. State space and waves.

3.4.8 *Supercritical regime*

As an upstream condition in subcritical regime (q_l does not exist), it would be valid because $q^\perp = q_m$. As an upstream supercritical regime boundary condition ($q_l = (h_{cc}, hu_{cc})$), it would be valid because $q^\perp = q_l$.

As a downstream boundary condition (inverse or subcritical), it would be wrong because $q^\perp \neq q_r$ in supercritical ($q_r = (h_{cc}, hu_{cc})$) and $q^\perp \neq q_m$ in subcritical.

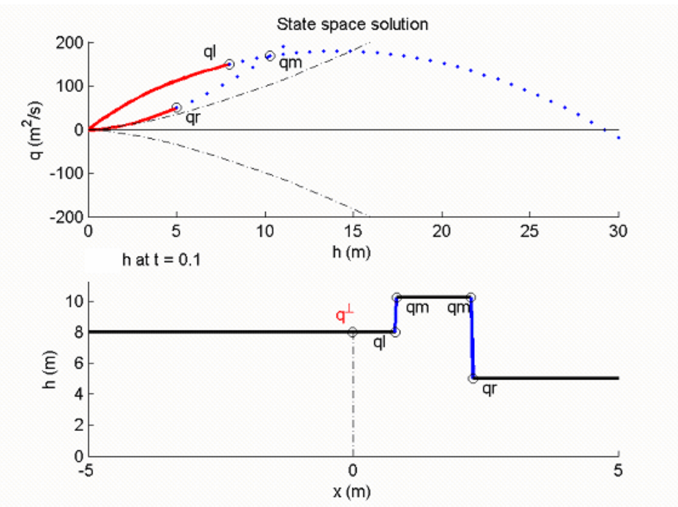

Figure 16. State space and waves

3.4.9 *Subcritical regime*

This case is important because, if we use $q_l = (h_{cc}, hu_{cc})$ as an upstream supercritical boundary condition, we find that $q^\perp \neq q_l$, although q_l is inside the supercritical zone. Thus, it is not a valid upstream supercritical condition.

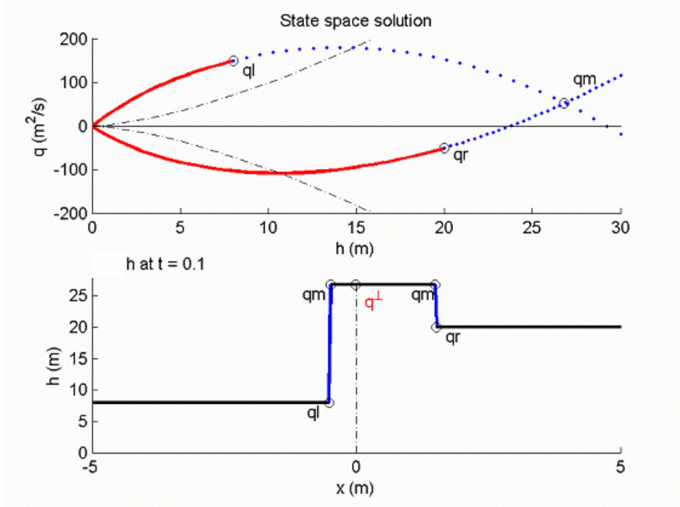

Figure 17. State space and waves

The condition that must be valid is $s > 0$ (Rankine-Hugoniot speed) in this case, the slope of the imaginary line joining q_l and q_m must be positive:

$$\frac{hu_l - hu_m}{h_l - h_m} > 0 \tag{10}$$

A similar condition must be imposed on the inverse regimes. In this case, the slope of the imaginary line joining q_r and q_m must be negative:

$$\frac{hu_r - hu_m}{h_r - h_m} < 0 \tag{11}$$

3.4.10 *Supercritical regime*

This case is the symmetric of 3.4.5, so the interpretation of the boundary conditions is symmetric.

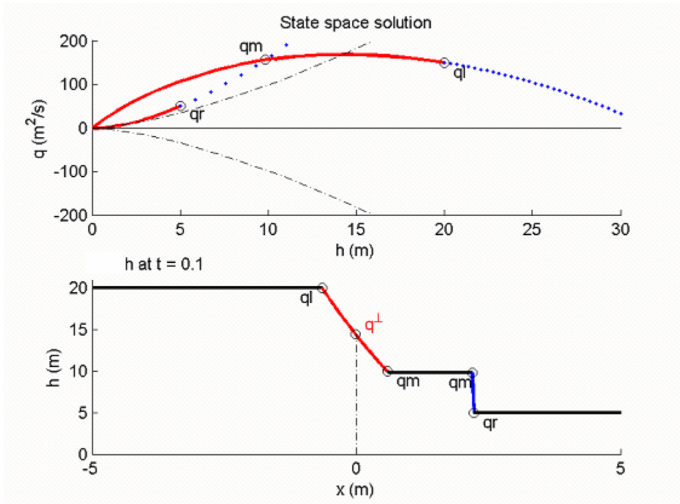

Figure 18. State space and waves.

3.5 *Solution algorithm*

The type of algorithm depends on the type of boundary condition to be imposed. It does not depend on the regime inside the domain.

3.5.1 *Supercritical regime:*

a) Check the imposed values of the dependent variables are included in the supercritical regime zone in the state space.
b) Solve the Riemann problem (q_l, q_r) and find q_m.
c) Check condition (10) to define $q^{\perp} = q_l$.

3.5.2 *Inverse regime:*

a) Check the imposed values of the dependent variables are included in the inverse regime zone in the state space.
b) Solve the Riemann problem (q_l, q_r) and find q_m.
c) Check condition (11) to define $q^{\perp} = q_t$.

3.5.3 *Subcritical regime:*

To define the subcritical regime algorithm, previous steps must be carried out. In this regime, we impose one boundary condition with a dependent variable (h_{cc} or hu_{cc}) and we interpolate the other dependent variable (hu_{cc} or h_{cc}). To interpolate the unknown dependent variables, we use the waves that connect the known state. In these waves, we find the state that has the imposed boundary condition value (Figure 5). However, the curves are not injective (the shapes are parabolic).

However, there is another limitation. The interpolated state must not be inside the supercritical or inverse regime zones, because if it were it would not be a subcritical condition. Excluding the zones that correspond to supercritical or inverse regimes, the remaining part of the curves is injective. Thus, the mathematical and hydraulic problems are coupled. The limit state is the intersection between the curves and the Crit+ and Crit- curves. This intersection defines the state (h_c, hu_{cc}). For an upstream condition, our boundary condition dependent variables must fulfill:

$$\begin{aligned} h_{cc} &> h_c \\ hu_{cc} &> hu_c \end{aligned} \tag{12}$$

The state is therefore outside of the inverse regime zone.

If we impose a downstream boundary condition, the dependent variables must fulfill:

$$\begin{aligned} h_{cc} &> h_c \\ hu_{cc} &< hu_c \end{aligned} \tag{13}$$

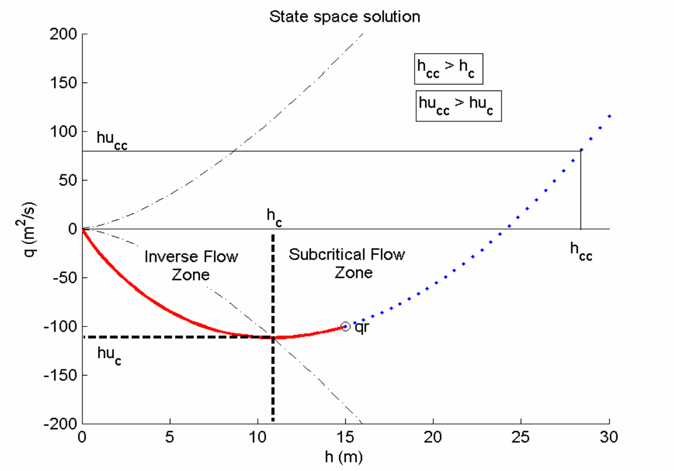

Figure 19 State limit that must be applied to the upstream boundary conditions of subcritical regimes

This limitation imposes a maximum for the negative discharges for upstream and downstream boundary conditions. The algorithm is as follows:

a) Determine the values of h_c and hu_c as interpolation limits.
b) Considering these limitations, find the unknown dependent variable that corresponds to the known dependent variable imposed as a boundary condition. The result of this step is q_m.
c) Check that state q_m is not in the supercritical or subcritical zone, because that would indicate a compatible but complete boundary condition (which is only valid for supercritical and inverse regimes).
d) At this point, we know that $q^{\perp} \in [q_m, q_r]$ or $q^{\perp} \in [q_m, q_l]$. We only need to check that $q_m = q^{\perp}$.

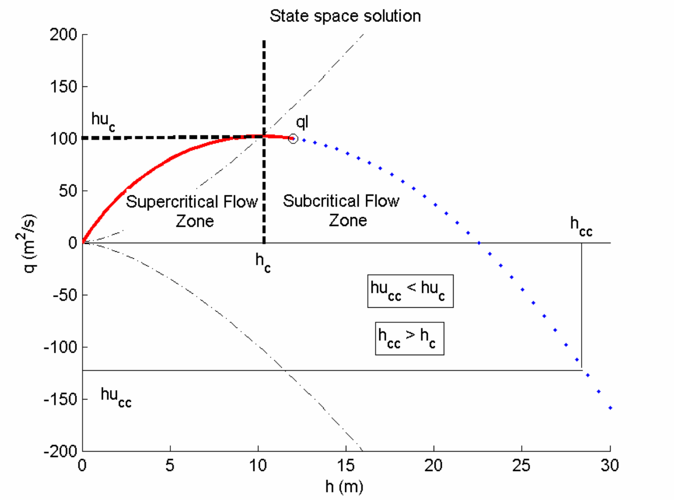

Figure 20 State limit that must be applied to the downstream boundary conditions of subcritical regimes

A sufficient condition to fulfill d) is that the inside state $(q_l \text{ or } q_r)$ is outside of the inverse and supercritical regime zones.

Figure 21 shows an example of this situation. If we impose a subcritical upstream boundary condition (q_m, q_r), q_m fulfills c) but the inside state q_r does not fulfill d). Finally, $q_l \neq q_m$, so it is an invalid upstream boundary condition.

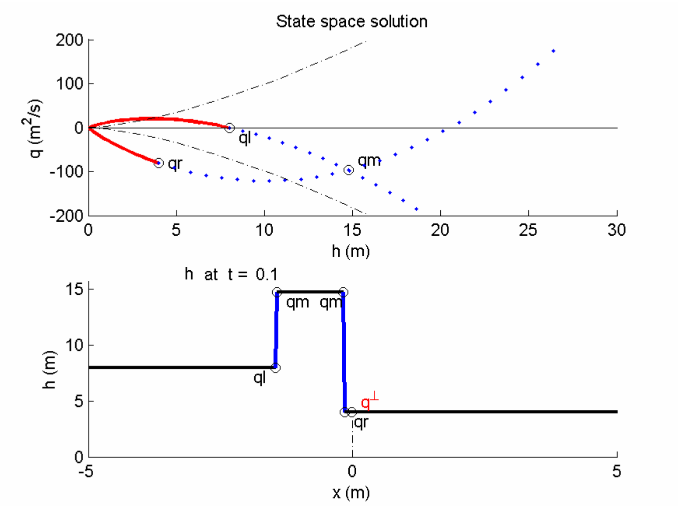

Figure 21. Example of a violation of subcritical boundary conditions

Let us conceptually interpret these limitations. Condition a) imposes that the boundary conditions at least have an influence inside the domain (upwind). Condition c) imposes that the condition may not be complete, because that would imply imposing two values for dependent variables, which is not valid for subcritical regimes.

In hydraulic terms, this condition means that neither the upstream nor downstream discharge may exceed the critical discharge in a channel.

4 RESULTS

This section presents results obtained in a simulation involving a channel 10 meters long and 1 meter wide. The initial conditions are rest, 1 meter depth. The boundary conditions are an upstream discharge of 1 cubic meter per second and a downstream water level of 2 meters.

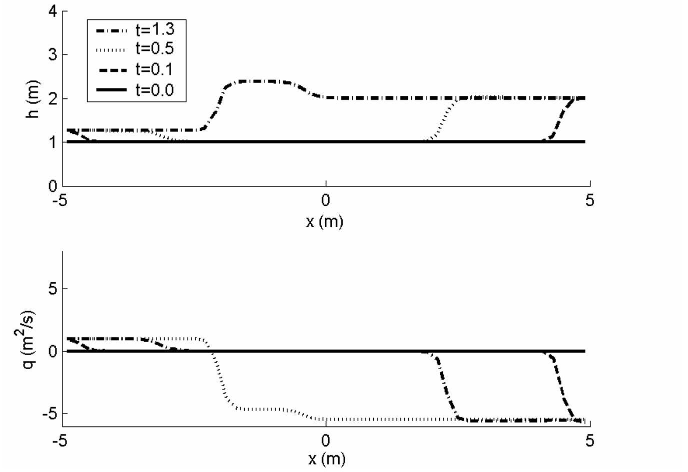

Figure 22. Four states of the channel in the first seconds

Figure 22 and Figure 23 show different time results.

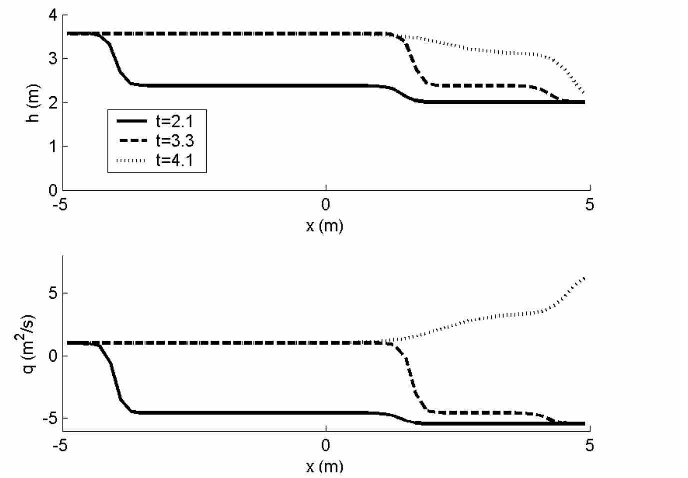

Figure 23. Three states of the channel in the first seconds

When the boundary conditions are imposed, shocks come from the boundaries, and keep unaltered until the wave traveling from the opposite node arrives. Figure 24 shows the changes over time in the depth and discharge in the upstream boundary and in the inside domain node next to the upstream boundary.

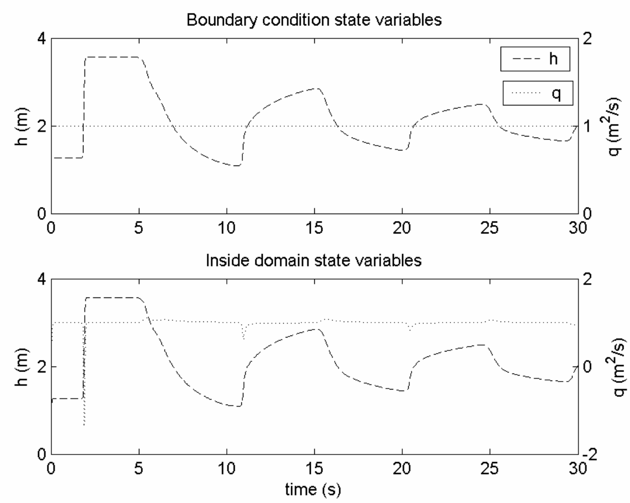

Figure 24. The dependent variables in the upstream boundary and in the inside domain node next to the upstream boundary

Figure 25 shows the changes over time in the state space in the upstream. The boundary curve is always at 1 cubic meter per second because it is imposed, and it changes the depth to adapt to the inside domain situation. The inside node line changes over time until it reaches a steady state, as does the whole channel, in the imposed boundary conditions, discharge 1 m^3/s and depth 2 m. Could happen that the imposed boundary conditions are wrong, in this situation the computation should stop or use spetial condition like critical depth.

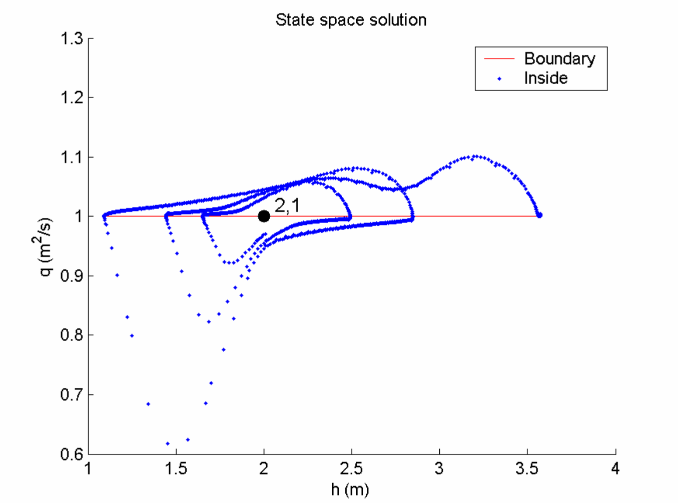

Figure 25. State space variables evolution in the upstream boundary condition and in the inside domain node, both are converging to boundary conditions, h=2 m , q=1 m^3/s

5 CONCLUSIONS

The proposed method combines a filter, which ensures that only well-posed problems are defined, and a method for interpolating the unknown dependent variables in partial boundary conditions (subcritical regimes).

The method is based on the Riemann problem solution and is optimal for the Godunov method.

It allows discontinuities such as boundary conditions to be introduced.

The method can immediately be extended to one-dimensional Euler equations, but more research is required before it can be extended to multidimensional problems.

The method is computed using an exact Riemann solver because is not computationally expensive, so the combination with approximate Riemann solvers should be researched.

6 ACKNOWLEDGMENTS.

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